

ON RADIATIVE $\phi \rightarrow \eta\gamma$, $\phi \rightarrow \eta'\gamma$ DECAYS*

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*Centre for Particle Theory, University of Durham, Durham DH1 3LE, U.K.***ABSTRACT**

We use QCD sum-rules to study the decays $\phi \rightarrow \eta\gamma$ and $\phi \rightarrow \eta'\gamma$, obtaining $\mathcal{B}(\phi \rightarrow \eta\gamma) = (1.15 \pm 0.2) 10^{-2}$ and $\mathcal{B}(\phi \rightarrow \eta'\gamma) = (1.18 \pm 0.4) 10^{-4}$, in very good agreement with existing experimental data. We also discuss the issue of η - η' mixing, predicting the η and η' decay constants in a mixing scheme in the quark-flavour basis.

1 Introduction

The reasons for interest in ϕ radiative decays are manifold. These decays yield information on low energy hadron physics, providing insight into the controversial issue of η - η' mixing. From the experimental side, new data are expected from the KLOE experiment at the DAΦNE ϕ -factory ¹⁾, where a large sample of ϕ decays will be collected, which will considerably improve the presently available statistics ²⁾.

In the following we study $\phi \rightarrow \eta\gamma$, $\phi \rightarrow \eta'\gamma$ decays using QCD sum-rules ³⁾. After surveying η - η' mixing, we consider the coupling of the strange pseudoscalar current to η and η' , preliminary to our analysis of ϕ decays. Though we work with interpolating currents defined in the quark flavour basis, our results do not depend on any mixing scheme, providing mixing scheme independent QCD predictions.

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We also consider the coupling of η and η' to the strange and non-strange axial currents, identifying the results with the decay constants in the flavour basis mixing scheme and estimating the mixing parameters. Finally, we give conclusions.

2 On $\eta - \eta'$ Mixing

$\eta - \eta'$ mixing is a much debated subject. The once conventional description introduced a single mixing angle in the octet-singlet flavour basis ⁴⁾. More recently, it has been shown ⁵⁾ that a proper treatment requires two angles with a redefinition of the particle decay constants. An equivalent description adopts the quark-flavour basis instead of the octet-singlet one ⁶⁾, in which such constants are defined as:

$$\langle 0 | J_{5\mu}^a | P(p) \rangle = i f_P^a p_\mu \quad (a = q, s; \quad P = \eta, \eta') , \quad (1)$$

with $J_{5\mu}^q = \frac{1}{\sqrt{2}}(\bar{u}\gamma_\mu\gamma_5 u + \bar{d}\gamma_\mu\gamma_5 d)$, $J_{5\mu}^s = \bar{s}\gamma_\mu\gamma_5 s$. The decay constants are written according to the following mixing pattern:

$$\begin{aligned} f_\eta^q &= f_q \cos \phi_q & f_\eta^s &= -f_s \sin \phi_s \\ f_{\eta'}^q &= f_q \sin \phi_q & f_{\eta'}^s &= f_s \cos \phi_s \quad . \end{aligned} \quad (2)$$

Feldmann ⁶⁾ has shown that, since $|\phi_s - \phi_q|/(\phi_s + \phi_q) \ll 1$, this mixing framework is simpler, being specified quite accurately in terms of a single angle $\phi = \phi_q = \phi_s$.

3 η and η' couplings to the pseudoscalar current

Let us define: $\langle 0 | \bar{s}i\gamma_5 s | \eta \rangle = A$, and compute this quantity using QCD sum-rules starting from the two-point correlator:

$$T_A(q^2) = i \int d^4x e^{iq \cdot x} \langle 0 | T[J_5^s(x) J_5^{s\dagger}(0)] | 0 \rangle \quad (3)$$

where $J_5^s = \bar{s}i\gamma_5 s$. The correlator (3) is given by the dispersive representation:

$$T_A(q^2) = \frac{1}{\pi} \int_{4m_s^2}^{\infty} ds \frac{\rho(s)}{s - q^2} + \text{subtractions} . \quad (4)$$

For low values of s , $\rho(s)$ receives contribution from the coupling of the η to the pseudoscalar current. Hence we can write (we discuss later possible subtractions):

$$T_A(q^2) = \frac{A^2}{m_\eta^2 - q^2} + \frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{\rho^{had}(s)}{s - q^2} , \quad (5)$$

assuming that higher contributions start from an effective threshold s_0 . $T_A(q^2)$ can also be computed in QCD by expanding the T -product in (3) by an Operator Product Expansion as the sum of a perturbative term plus non-perturbative ones proportional to vacuum condensates. Dispersively writing the perturbative term too, we have:

$$T_A^{QCD}(q^2) = \frac{1}{\pi} \int_{4m_s^2}^{\infty} ds \frac{\rho^{QCD}(s)}{s - q^2} + d_3 < \bar{s}s > + d_5 < \bar{s}g\sigma Gs > + \dots \quad (6)$$

ρ^{QCD} and d_3, d_5 are computed in QCD. Now we implement quark-hadron duality, assuming that $\rho^{had}(s)$ and $\rho^{QCD}(s)$ give the same result when integrated above s_0 . This leads to the sum-rule:

$$\frac{A^2}{m_\eta^2 - q^2} = \frac{1}{\pi} \int_{4m_s^2}^{s_0} ds \frac{\rho^{QCD}(s)}{s - q^2} + d_3 < \bar{s}s > + d_5 < \bar{s}g\sigma Gs > + \dots \quad (7)$$

Now we apply to both sides of (7) a Borel transform, defined as

$$\mathcal{B}[f(Q^2)] = \lim_{Q^2 \rightarrow \infty, n \rightarrow \infty, \frac{Q^2}{n} = M^2} \frac{1}{(n-1)!} (-Q^2)^n \left(\frac{d}{dQ^2} \right)^n f(Q^2), \quad (8)$$

where f is a generic function of $Q^2 = -q^2$ and M^2 is known as the Borel parameter. This operation improves the convergence of the sum rule and eliminates the contribution of subtraction terms in (4). The final sum-rule reads:

$$A^2 e^{-\frac{m_\eta^2}{M^2}} = \frac{3}{8\pi^2} \int_{4m_s^2}^{s_0} ds s \sqrt{1 - \frac{4m_s^2}{s}} e^{-\frac{s}{M^2}} - m_s e^{-\frac{m_s^2}{M^2}} \left[< \bar{s}s > \left(1 - \frac{m_s^2}{M^2} + \frac{m_s^4}{M^4} \right) + \frac{1}{M^2} < \bar{s}g\sigma Gs > \left(1 - \frac{m_s^2}{2M^2} \right) \right]. \quad (9)$$

We use $< \bar{s}g\sigma Gs > = 0.8 \text{ GeV}^2 < \bar{s}s >$, $< \bar{s}s > = 0.8 < \bar{q}q >$, $< \bar{q}q > = (-0.24)^3 \text{ GeV}^3$, $m_\eta = 0.548 \text{ GeV}$. We vary the strange quark mass in the range: $m_s = 0.125 - 0.140 \text{ GeV}$ ⁷⁾ and s_0 below the η' pole between $0.9^2 - 0.95^2 \text{ GeV}^2$. Since M^2 is an unphysical parameter, we look for a range of its values (“stability window”) where the sum-rule is almost independent of M^2 . We fix M^2 in $[0.8, 1] \text{ GeV}^2$ and, considering the uncertainty on m_s and on s_0 , we get ⁸⁾:

$$|A| = (0.115 \pm 0.004) \text{ GeV}^2. \quad (10)$$

Let us now consider: $< 0 | \bar{s} i \gamma_5 s | \eta' > = A'$. An analogous calculation gives:

$$(A')^2 e^{-\frac{m_{\eta'}^2}{M^2}} + A^2 e^{-\frac{m_\eta^2}{M^2}} = \frac{3}{8\pi^2} \int_{4m_s^2}^{s'_0} ds s \sqrt{1 - \frac{4m_s^2}{s}} e^{-\frac{s}{M^2}} - m_s e^{-\frac{m_s^2}{M^2}} \left[< \bar{s}s > \left(1 - \frac{m_s^2}{M^2} + \frac{m_s^4}{M^4} \right) + \frac{1}{M^2} < \bar{s}g\sigma Gs > \left(1 - \frac{m_s^2}{2M^2} \right) \right], \quad (11)$$

where we have raised the threshold up to s'_0 as to pick up the η' pole too. Using $m_{\eta'} = 0.958$ GeV and in the stability window $[1.2, 2]$ GeV² for M^2 , we obtain ⁸⁾:

$$|A'| = (0.151 \pm 0.015) \text{ GeV}^2. \quad (12)$$

Though we cannot actually establish the sign of A , A' , we assume that $A \cdot A' > 0$.

4 Radiative $\phi \rightarrow \eta\gamma$ and $\phi \rightarrow \eta'\gamma$ decays

Let us define: $\langle \eta(q_2) | \bar{s}\gamma^\nu s | \phi(q_1, \epsilon_1) \rangle = F(q^2) \epsilon^{\nu\alpha\beta\delta} (q_1)_\alpha (q_2)_\beta (\epsilon_1)_\delta$, ($q = q_1 - q_2$). In order to compute $\phi \rightarrow \eta\gamma$ decay, we need the coupling $g = -\frac{1}{3}F(0)$, obtained for a real photon coupling to a strange quark. We consider:

$$\Pi_{\mu\nu} = i^2 \int d^4x d^4y e^{-iq_1 \cdot x + iq_2 \cdot y} \langle 0 | T[J_5^s(y) J_\nu(0) J_\mu(x)] | 0 \rangle = \Pi(q_1^2, q_2^2, q^2) \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta \quad (13)$$

with J_5^s defined above and $J_\nu = \bar{s}\gamma_\nu s$. The sum-rule is built up for $\Pi(q_1^2, q_2^2, q^2)$. First we write $\Pi(q_1^2, q_2^2, q^2)$ according to a dispersion relation in the variables q_1^2, q_2^2 :

$$\Pi(q_1^2, q_2^2, q^2) = \frac{1}{\pi^2} \int ds_1 \int ds_2 \frac{\rho(s_1, s_2, q^2)}{(s_1 - q_1^2)(s_2 - q_2^2)}. \quad (14)$$

Now the spectral function contains, for low values of s_1, s_2 , a double δ -function corresponding to the transition $\phi \rightarrow \eta$. Extracting this contribution, we derive the sum-rule (after a double Borel transform in the variables $-q_1^2, -q_2^2$):

$$\begin{aligned} A F(q^2) m_\phi f_\phi &= e^{\frac{m_\phi^2}{M_1^2}} e^{\frac{m_\eta^2}{M_2^2}} \left\{ \int ds_1 \int ds_2 e^{-\frac{s_1}{M_1^2}} e^{-\frac{s_2}{M_2^2}} \frac{3m_s}{\pi^2 \sqrt{\lambda(s_1, s_2, q^2)}} \right. \\ &+ e^{-\frac{m_s^2}{M_1^2}} e^{-\frac{m_s^2}{M_2^2}} \left[\langle \bar{s}s \rangle \left(2 - \frac{m_s^2}{M_1^2} - \frac{m_s^2}{M_2^2} + \frac{m_s^4}{M_1^4} + \frac{m_s^4}{M_2^4} + \frac{m_s^2(2m_s^2 - q^2)}{M_1^2 M_2^2} \right) \right. \\ &\left. \left. + \langle \bar{s}g\sigma Gs \rangle \left(\frac{1}{6M_1^2} + \frac{2}{3M_2^2} - \frac{m_s^2}{2M_1^4} - \frac{m_s^2}{2M_2^4} + \frac{(2q^2 - 3m_s^2)}{3M_1^2 M_2^2} \right) \right] \right\} \quad (15) \end{aligned}$$

The integration domain over s_1, s_2 (specified in ⁸⁾) depends on q^2 . We compute $F(q^2)$ for negative values of q^2 , and then extrapolate the result to $q^2 = 0$ ¹. Since we only know the magnitude of A , it is $|F(q^2)|$ that is determined. We use: $m_\phi = 1.02$ GeV, $f_\phi = 0.234$ GeV (obtained from the experimental datum on $\phi \rightarrow e^+e^-$ ⁹⁾). We use two values of the ϕ threshold: $s_{01} = 1.8, 1.9$ GeV²; the η threshold is chosen as in section 3. The extrapolation to $q^2 = 0$ gives $|g| = F(0)/3 = (0.66 \pm 0.06) \text{ GeV}^{-1}$;

¹In this way, we could perform a double Borel transform in the two variables $Q_1^2 = -q_1^2$ and $Q_2^2 = -q_2^2$, which allows us to remove single poles in the s_1 and s_2 channels (“parasitic” terms).

the uncertainty is obtained varying all the input parameters in the sum rule. From this result we compute $\Gamma(\phi \rightarrow \eta\gamma)$ and, using $\Gamma(\phi) = 4.43 \text{ MeV}$ ⁹⁾, find ⁸⁾:

$$\mathcal{B}(\phi \rightarrow \eta\gamma) = (1.15 \pm 0.2)\% , \quad (16)$$

in agreement with the experimental result: $\mathcal{B}(\phi \rightarrow \eta\gamma) = (1.18 \pm 0.03 \pm 0.06)\%$ ²⁾. Applying the same analysis to the η' mode gives $|g'| = F'(0)/3 = (1.0 \pm 0.2) \text{ GeV}^{-1}$ and ⁸⁾

$$\mathcal{B}(\phi \rightarrow \eta'\gamma) = (1.18 \pm 0.4) 10^{-4} , \quad (17)$$

which agrees with the experimental datum $\mathcal{B}(\phi \rightarrow \eta'\gamma) = (0.82_{-0.19}^{+0.21} \pm 0.11) 10^{-4}$ ²⁾. The results (16) and (17) are independent of any mixing scheme for the η and η' . Nevertheless, adopting the mixing scheme in the flavour basis, one gets:

$$R = \frac{\mathcal{B}(\phi \rightarrow \eta\gamma)}{\mathcal{B}(\phi \rightarrow \eta'\gamma)} = \left(\frac{m_\phi^2 - m_\eta^2}{m_\phi^2 - m_{\eta'}^2} \right)^3 \tan^2 \phi_s \quad (18)$$

and hence $\phi_s = (34 \pm 8)^\circ$. The experimental ratio would give: $\phi_s = (39.0 \pm 7.5)^\circ$. For a comprehensive survey of other results, we refer to ⁴⁾.

5 η and η' couplings to axial currents

Now we investigate the η , η' decay constants in both the strange and non-strange sectors in order to deduce the mixing pattern. We begin by considering the correlator

$$\Pi_{\mu\nu}(q^2) = i \int d^4x e^{iq \cdot x} \langle 0 | T [J_{5\mu}^s(x) J_{5\nu}^s(0)] | 0 \rangle . \quad (19)$$

Following the procedure already outlined above, we obtain the sum-rule:

$$(f_\eta^s)^2 = e^{\frac{m_\eta^2}{M^2}} \left[\frac{1}{4\pi^2} \int_{4m_s^2}^{s_0} ds e^{-\frac{s}{M^2}} \sqrt{1 - \frac{4m_s^2}{s}} \frac{2m_s^2 + s}{s} + \frac{2m_s}{M^2} \langle \bar{s}s \rangle e^{-\frac{m_s^2}{M^2}} \right] . \quad (20)$$

In the stability window $[2, 3.5] \text{ GeV}^2$ for M^2 we get ⁸⁾ $f_\eta^s = (0.13 \pm 0.01) \text{ GeV}$ and, using the same technique: $f_{\eta'}^s = (0.12 \pm 0.02) \text{ GeV}$, $f_\eta^q = (0.144 \pm 0.004) \text{ GeV}$, $f_{\eta'}^q = (0.125 \pm 0.015) \text{ GeV}$. From these results we can estimate all the mixing parameters defined in (2), obtaining: $\phi_s = (46.6^\circ \pm 7^\circ)$, $f_s = (0.178 \pm 0.004) \text{ GeV}$ and $\phi_q = (41^\circ \pm 4^\circ)$, $f_q = (0.19 \pm 0.015) \text{ GeV}$. Our results give $|\phi_s - \phi_q|/(\phi_s + \phi_q) \simeq 0.065$, confirming that this ratio is much less than 1 ⁶⁾.

Finally, let us consider the matrix element: $\langle 0 | \partial^\mu J_{5\mu}^s | \eta \rangle = m_\eta^2 f_\eta^s$. The divergence of the axial current contains the anomaly: $\partial^\mu J_{5\mu}^s = \partial^\mu (\bar{s}\gamma_\mu \gamma_5 s) = 2m_s \bar{s}i\gamma_5 s + \frac{\alpha_s}{4\pi} G\tilde{G}$, G being the gluon field strength tensor, \tilde{G} its dual. Therefore:

$$2m_s \langle 0 | \bar{s}i\gamma_5 s | \eta^{(\prime)} \rangle = f_{\eta^{(\prime)}}^s m_{\eta^{(\prime)}}^2 - \langle 0 | \frac{\alpha_s}{4\pi} G\tilde{G} | \eta^{(\prime)} \rangle . \quad (21)$$

Exploiting the previous results and (10), (12), we derive from (21):

$$\langle 0 \left| \frac{\alpha_s}{4\pi} G\tilde{G} \right| \eta \rangle = (0.008 \pm 0.004) \text{ GeV}^3, \quad \langle 0 \left| \frac{\alpha_s}{4\pi} G\tilde{G} \right| \eta' \rangle = (0.072 \pm 0.025) \text{ GeV}^3. \quad (22)$$

Both values in (22) are close to the quark model result of Novikov et al.¹⁰⁾.

6 Conclusions

We have analysed $\phi \rightarrow \eta\gamma$, $\phi \rightarrow \eta'\gamma$ decays using QCD sum-rules. We begin by presenting a preliminary calculation of the couplings of the pseudoscalar current to η , η' . The results derived require no assumption about $\eta - \eta'$ mixing and are in good agreement with available data. However, the uncertainty in the η' case is large, and so the last word is left to the experimental improvement at DAΦNE, for instance.

We have also considered $\eta - \eta'$ mixing, estimating the mixing parameters in a quark-flavour basis scheme. The existing spread of results gives us confidence that new experimental information will shed light on this topic too.

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